Chapter # 3 Salient-Pole Synchronous Generator and Motor

1- Introduction

A cylindrical rotor synchronous machine has a uniform air-gap, therefore its reactance remains the same, irrespective of the rotor position. However, a synchronous machine with salient poles has non-uniform air-gap. Therefore, its reactance varies with the rotor position. Consequently, a cylindrical rotor machine possesses one axis of symmetry (pole axis or direct axis) whereas salient-pole machine possesses two axes of symmetry:

(i) field poles axis, called direct axis or *d*-axis and,

(ii) axis passing through the centre of the interpolar space, called the quadrature axis or *q*axis, as shown in Fig. 1.

Obviously, two mmfs act on the *d*-axis of a salient-pole synchronous machine *i.e.* field m.m.f. and armature m.m.f. whereas only one m.m.f., *i.e.* armature mmf acts on the *q*-axis, because field mmf has no component in the *q*-axis. The magnetic reluctance is low along the d-axis and it is high along the q-axis.

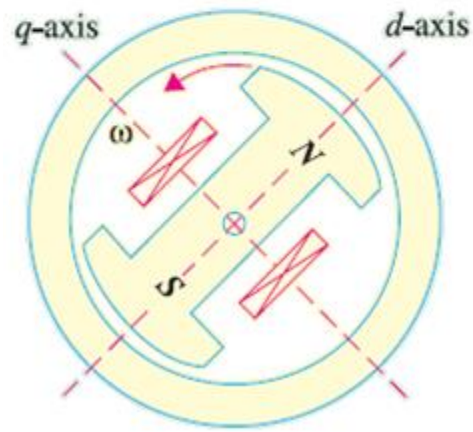
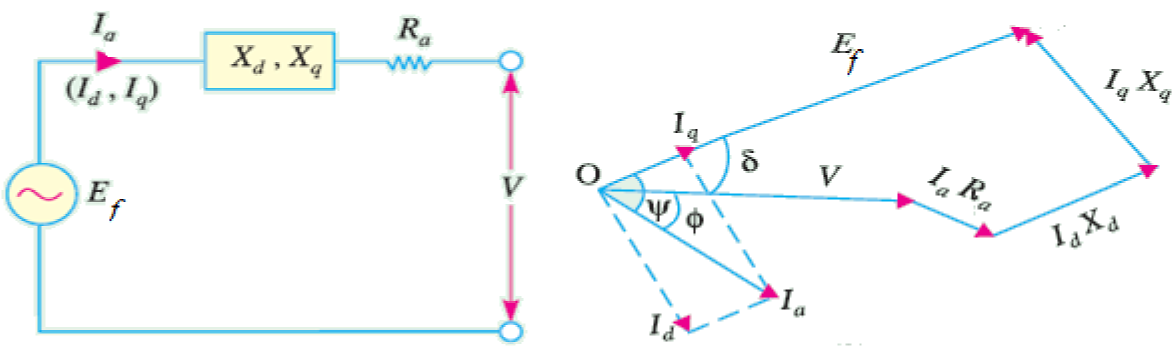


Fig. 1, Direct and Quadrature axes

The equivalent circuit and phasor diagram of a salient pole alternator is shown in Fig. 2.



(a) Equivalent Circuit (b) Phasor Diagram

Fig. 2, Salient-Pole synchronous generator

The above facts form the basis of the two-reaction theory proposed by Blondel, according to which

(*i*) armature current I_a can be resolved into two components I_{ad} perpendicular to E_f and I_{aq} along E_f as shown in Fig. 2 (*b*).

(*ii*) armature reactance has two components *i.e.* *q*-axis armature reactance X_{aq} and *d*-axis armature reactance X_{ad}.

If we include the armature leakage reactance X_L which is the same on both axes, we get the two components of the synchronous reactance in the d- and q-axes:

$$X_{sd} = X_{ad} + X_L \;\; and \; X_{sq} = X_{aq} + X_L$$

$$\vec{E}_f = \vec{I}_a R_a + J\vec{I}_a X_L + J\vec{I}_{ad} X_{ad} + J\vec{I}_{aq} X_{aq}$$

$$\vec{E}_f = \vec{I}_a R_a + J\vec{I}_{ad}(X_L + X_{ad}) + J\vec{I}_{aq}(X_L + X_{aq})$$

$$\vec{E}_f = \vec{I}_a R_a + J\vec{I}_{ad} X_{Sd} + J\vec{I}_{aq} X_{Sq}$$

The last equation is represented in the phasor diagram of unsaturated machine shown in Fig. 2 (b), knowing that I_aR_a // I_a & $I_a X_L \perp I_a$ & $I_d X_{ad} \perp I_d$ & $I_q X_{aq} \perp I_q$

2. Phasor Diagram of Unsaturated Generator when σ is unknown

Since R_a is neglected, this phasor diagram is approximate.

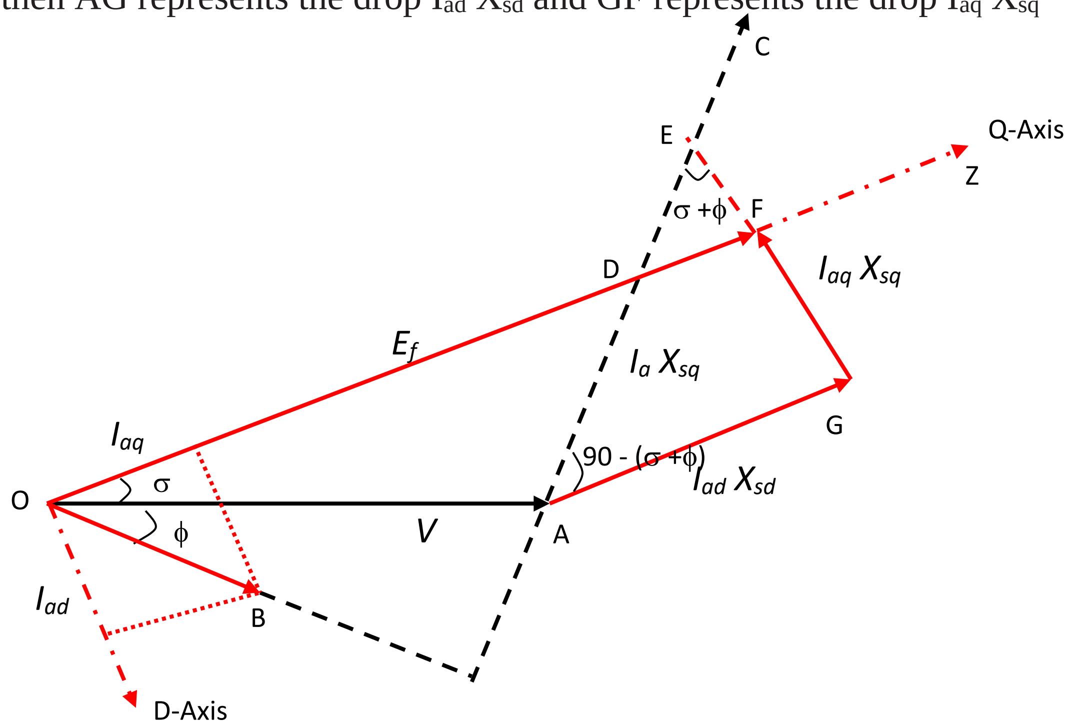
step 1: from V draw the line AC perpendicular to the current OB

Step 2: on the line AC locate the points D and E such that AD represents the drop $I_a X_{sq}$ and AE represents the drop $I_a X_{sd}$

Step 3: determine the Q-axis by drawing the line OZ passing through the point D

Step 4: determine the value of E_f represented by OF by drawing EF perpendicular to OZ

step 5: then AG represents the drop $I_{ad} X_{sd}$ and GF represents the drop $I_{aq} X_{sq}$



Some basic information from the above phasor diagram (P.f. Lagging):

Direct Axis Analysis

$$I_{ad} = I_a \sin(\delta + \varphi) \ldots.(1)$$
$$I_{aq}X_{sq} = V \sin(\delta) \ldots.(2)$$

Quadrature Axis Analysis

$$I_{aq} = I_a \cos(\delta + \varphi) \ldots(3)$$
$$E_f = V \cos(\delta) + I_{ad}X_{sd} \ldots(4)$$

substitute from (3) into (2)

$$V \sin(\delta) = I_a X_{sq} \cos(\delta + \varphi)$$
$$V \sin(\delta) = I_a X_{sq} (\cos\delta \, cos\varphi - \sin\delta \sin\varphi)$$

dividing both sides over cos(δ)

$$V \tan\delta = I_a X_{sq} (\cos\varphi - \tan\delta \sin\varphi)$$
$$\tan\delta (V + I_a X_{sq} \sin\varphi) = I_a X_{sq} \cos\varphi$$
$$\tan\delta \;\; = \frac{I_a X_{sq} \cos\varphi}{V + I_a X_{sq} \sin\varphi}$$

Once the load angle δ is determined, E_f can be obtained from (4)

Example (1):

*A 3-phase alternator has a direct-axis synchronous reactance of 0.7 p.u. and a quadrature axis synchronous reactance of 0.4 p.u. Draw the vector diagram for full-load 0.8 p.f. lagging and obtain **(i)** the load angle and **(ii)** the no-load per unit voltage.*

V = 1 p.u.; X_{sd} = 0.7 p.u.; X_{sq} = 0.4 p.u.;
cos φ = 0.8; sin φ = 0.6 ; φ = cos−1 0.8 = 36.9°; I_a = 1 p.u.

$$\tan\delta \;\; = \frac{I_a X_{sq} \cos\varphi}{V + I_a X_{sq} \sin\varphi} = \frac{1 \times 0.4 \times 0.8}{1 + 1 \times 0.4 \times 0.6} = 0.2581$$

δ = 14.47°

$$I_{ad} = I_a \sin(\delta + \varphi) = 1 \times \sin(14.47 + 36.9) = 0.78 \, p.u.$$
$$E_f = V \cos(\delta) + I_{ad}X_{sd} = 1 \times \cos(14.47) + 0.78 \times 0.7 = 1.514 \, p.u.$$

Example (2):

A 3-phase, Y-connected syn. generator supplies current of 10 A having phase angle of 20° lagging at phase voltage of 400 V. Find the load angle and the components of armature current I_d and I_q if X_{sd} = 10 ohm and X_{sq} = 6.5 ohm. Assume that R_a to be negligible.

Since R_a is neglected,

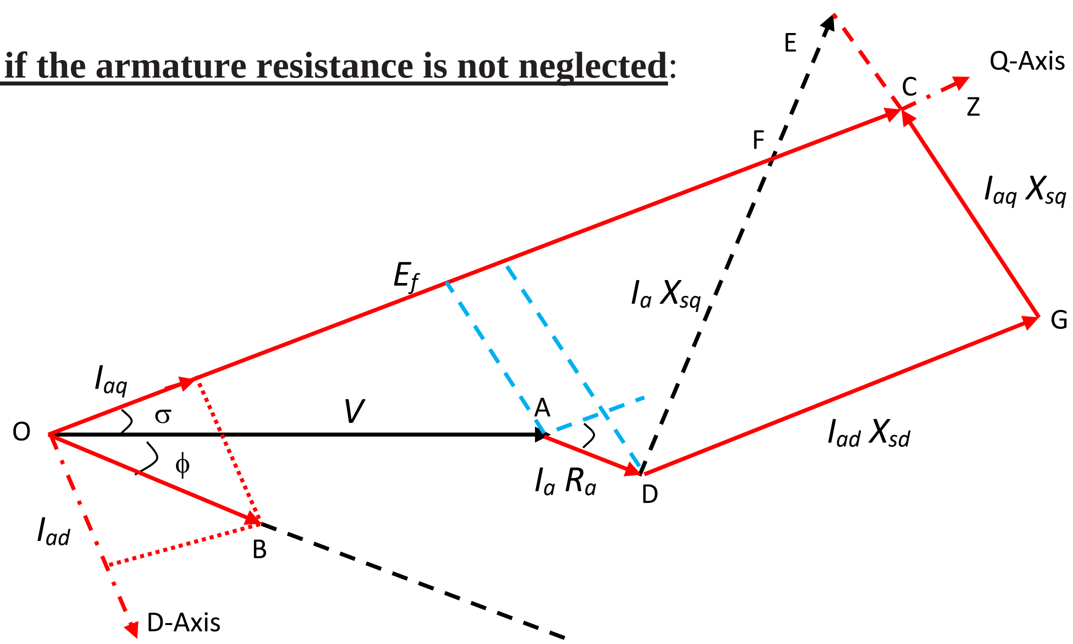
$\cos(\varphi) = 0.9397$, $\sin(\varphi) = 0.342$,

$$\tan\delta = \frac{I_a X_{sq}\cos\varphi}{V + I_a X_{sq}\sin\varphi} = \frac{10 \times 6.5 \times 0.9397}{400 + 10 \times 6.5 \times 0.342} = 0.1447$$

$\delta = 8.23°$

$$I_{ad} = I_a \sin(\delta+\varphi) = 10\sin(8.23+20) = 4.73\ A$$
$$I_{aq} = I_a \cos(\delta+\varphi) = 10\cos(8.23+20) = 8.811\ A$$
$$E_f = V\cos(\delta) + I_{ad}X_{sd} = 400 \times \cos(8.23) + 4.73 \times 10 = 443.181\ V$$
$$\in = \frac{E_f - V}{V} = \frac{443.181 - 400}{400} = 10.8\%\ (up)$$

<u>Now, if the armature resistance is not neglected</u>:



step 1: from V draw the line AD parallel to the current OB to represent I_aR_a

step 2: at D draw the line DE perpendicular to the current OB

Step 3: on the line DE locate the points F and E such that DF represents the drop $I_a\ X_{sq}$ and DE represents the drop $I_a\ X_{sd}$

Step 4: determine the Q-axis by drawing the line OZ passing through the point F

Step 5: determine the value of E_f represented by OC by drawing EC perpendicular to OZ

step 6: then DG represents the drop $I_{ad}\ X_{sd}$ and GC represents the drop $I_{aq}\ X_{sq}$

<u>Direct Axis Analysis</u>

$$I_{ad} = I_a \sin(\delta+\varphi) \dots .(1)$$
$$I_{aq}X_{sq} = V\sin(\delta) + I_aR_a\sin(\delta+\varphi) \dots .(2)$$

Quadrature Axis Analysis

$$I_{aq} = I_a \cos(\delta + \varphi) \dots (3)$$

$$E_f = V\cos(\delta) + I_a R_a \cos(\delta + \varphi) + I_{ad} X_{sd} \dots (4)$$

substitute from (3) into (2)

$$I_a \cos(\delta + \varphi) X_{sq} = V\sin(\delta) + I_a R_a \sin(\delta + \varphi)$$

$$I_a X_{sq}(\cos\delta\, cos\varphi - \sin\delta \sin\varphi) = V\sin(\delta) + I_a R_a(\sin\delta\, cos\varphi + \cos\delta \sin\varphi)$$

dividing both sides over cos(δ)

$$I_a X_{sq}(cos\varphi\ - \tan\delta \sin\varphi) = V\tan(\delta) + I_a R_a(\tan\delta\, cos\varphi + \sin\varphi)$$

$$\tan\delta(V + I_a X_{sq}\sin\varphi + I_a R_a cos\varphi) = I_a X_{sq}\cos\varphi - I_a R_a \sin\varphi$$

$$\tan\delta = \frac{I_a X_{sq}\cos\varphi - I_a R_a \sin\varphi}{V + I_a X_{sq}\sin\varphi + I_a R_a cos\varphi}$$

Once the load angle δ is determined, E_f can be obtained from (4)

Example (3):

A 3-phase, star-connected, 50-Hz synchronous generator has direct-axis synchronous reactance of 0.6 p.u. and quadrature-axis synchronous reactance of 0.45 p.u. The generator delivers rated kVA at rated voltage. Draw the phasor diagram at full-load 0.8 p.f. lagging and hence calculate the open-circuit voltage and voltage regulation. Resistive drop at full-load is 0.015 p.u.

I_a = 1 p.u.; V = 1 p.u.; X_{sd} = 0.6 p.u.; X_{sq} = 0.45 p.u. ; R_a = 0.015 p.u.
cos φ = 0.8; sin φ = 0.6 ; φ = cos−1 0.8 = 36.9°

$$\tan\delta = \frac{I_a X_{sq}\cos\varphi - I_a R_a \sin\varphi}{V + I_a X_{sq}\sin\varphi + I_a R_a cos\varphi} = \frac{1\times 0.45\times 0.8 - 1\times 0.015\times 0.6}{1 + 1\times 0.45\times 0.6 + 1\times 0.015\times 0.8} = 0.274$$

Then δ = 15.3°

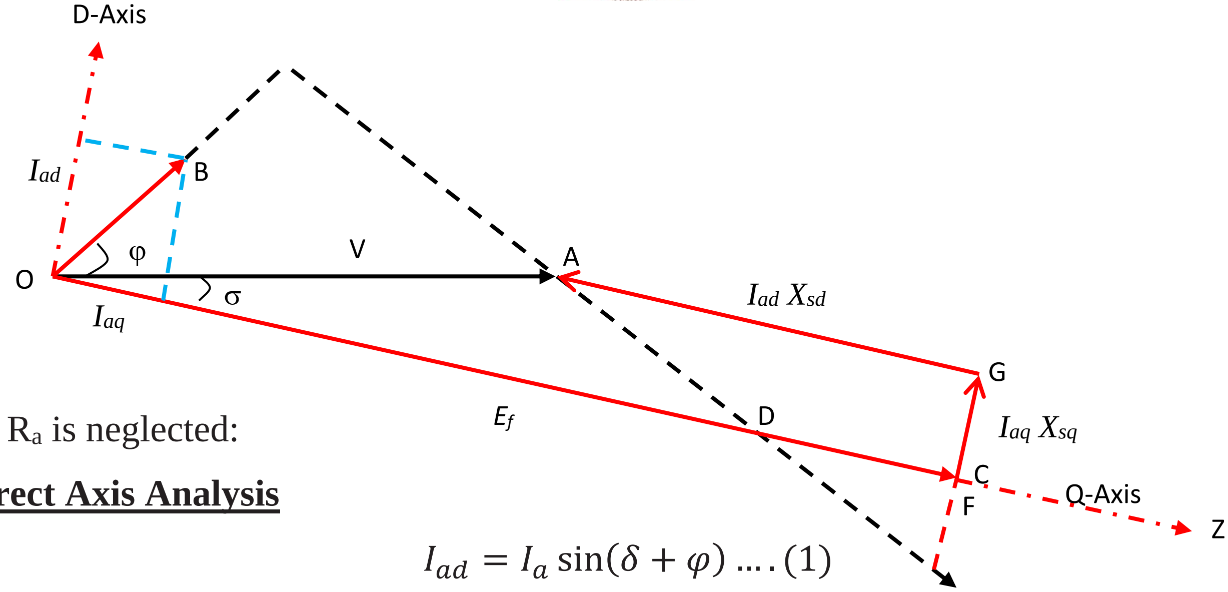
$$I_{ad} = I_a \sin(\delta + \varphi) = 1\times\sin(15.3 + 36.9) = 0.79\ p.u$$

$$E_f = V\cos(\delta) + I_a R_a \cos(\delta + \varphi) + I_{ad} X_{sd}$$

$$= 1\times\cos(15.3) + 1\times 0.015\times\cos(52.2) + 0.79\times 0.6 = 1.45\ p.u$$

$$\epsilon = \frac{E_f - V}{V} = \frac{1.45 - 1}{1} = 45\%\ (up)$$

For synchronous motor, the phasor diagram can be drawn as follows:



As R_a is neglected:

<u>Direct Axis Analysis</u>

$$I_{ad} = I_a \sin(\delta + \varphi) \;\ldots.(1)$$
$$I_{aq}X_{sq} = V\sin(\delta) \;\ldots.(2)$$

<u>Quadrature Axis Analysis</u>

$$I_{aq} = I_a \cos(\delta + \varphi) \;\ldots(3)$$
$$E_f = V\cos(\delta) + \; I_{ad}X_{sd} \;\ldots(4)$$

substitute from (3) into (2)

$$V\sin(\delta) = I_a X_{sq}\cos(\delta + \varphi)$$
$$V\sin(\delta) = I_a X_{sq}(\cos\delta\; cos\varphi - \sin\delta\sin\varphi)$$

dividing both sides over cos(δ)

$$V\tan\delta = I_a X_{sq}(\cos\varphi - \tan\delta\sin\varphi)$$
$$\tan\delta(V + I_a X_{sq}\sin\varphi) = I_a X_{sq}\cos\varphi$$

$$\tan\delta \;=\; \frac{I_a X_{sq}\cos\varphi}{V + I_a X_{sq}\sin\varphi}$$

Once the load angle δ is determined, E_f can be obtained from (4)

<u>This case of Motor lead P.F. is similar to the case of Generator with Lag P.F.</u>

2. Power Developed by a synchronous Generator

If we neglect R_a the phasor diagram of salient pole generator is as shown in Fig. 4, and hence the Cu loss, then the power developed (P_d) or the electromagnetic power (P_{em}) by an alternator is equal to the power output (P_{out}). Hence, the per phase power output of an alternator is

$P_{out} = VI_a \cos(\varphi)$ = power developed (pd) ...(***i***)

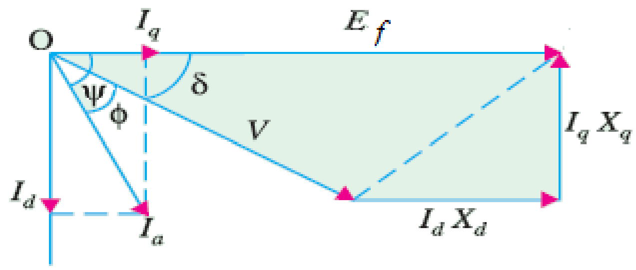


Fig. 4,

Now, as seen from Fig. 3,

$$I_q X_{Sq} = V \sin \delta \; ; I_d X_{Sd} = E_f - V \cos \delta \; ...(\textbf{\textit{ii}})$$

Also,

$$I_d = I_a \sin(\varphi + \delta); I_q = I_a \cos(\varphi + \delta) \; ...(\textbf{\textit{iii}})$$

Substituting Eqn. (*iii*) in Eqn. (*ii*) and solving for $I_a \cos\varphi$, we get

$$I_a \cos(\phi) = \frac{E_f}{X_{Sd}} \sin(\delta) + \frac{V}{2X_{Sq}} \sin(2\delta) - \frac{V}{2X_{Sd}} \sin(2\delta)$$

Finally, substituting the above in Eqn. (*i*), we get the 3-ph power

$$P_{out} = \frac{3\,VE_f}{X_{Sd}} \sin(\delta) + \frac{3\,V^2(X_{Sd} - X_{Sq})}{2\,X_{Sd}\,X_{Sq}}\; Sin(2\delta)$$

As seen from the above expression and its representation shown in Fig. 5, the power developed consists of two components, the first term represents the power due to excitation and depend on E_f and the second term gives the reluctance power *i.e.* power due to saliency. If $X_{Sd} = X_{Sq}$ i.e. the machine has a cylindrical rotor, then the second term becomes zero and the power is given by the first term only. If, on the other hand, there is no field excitation i.e. $E_f = 0$, then the first term in the above expression becomes zero and the power developed is given by the second term. It may be noted that value of δ is positive for a generator and negative for a motor.

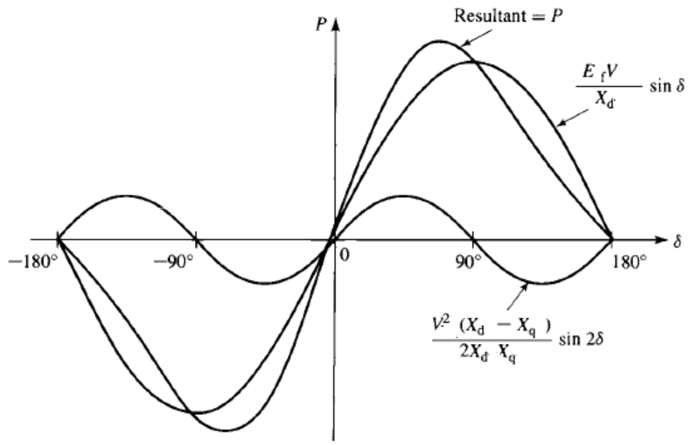
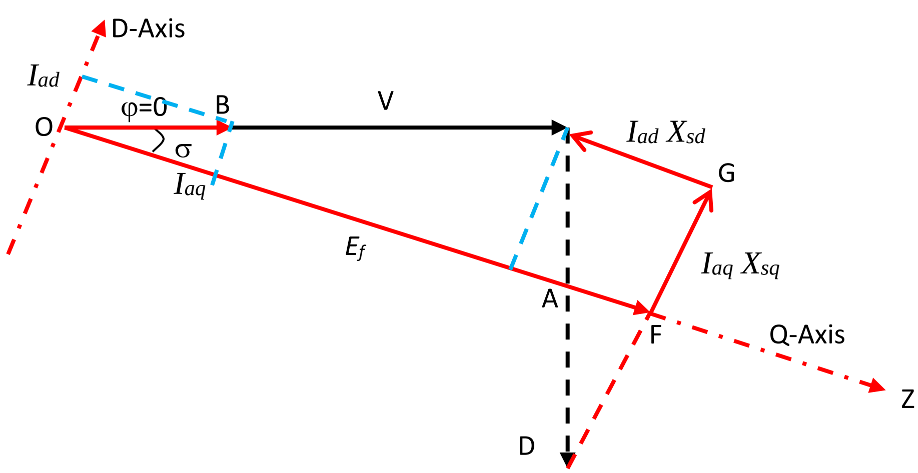


Fig. 5, power in salient-pole synchronous machine

Example (4):

A 3300-V, 1.5-MW, 3-φ, Y-connected synchronous motor has X_{sd} = 4Ω / phase and X_{sq} = 3 Ω / phase. Neglecting all losses, calculate the excitation e.m.f. when motor supplies rated load at unity p.f. Calculate the maximum mechanical power which the motor would develop for this field excitation.

The phasor diagram of synchronous motor at unity p.f. is given below:



Direct Axis Analysis

$$I_{ad} = I_a \sin(\delta) \dots (1)$$
$$I_{aq} X_{sq} = V \sin(\delta) \dots (2)$$

Quadrature Axis Analysis

$$I_{aq} = I_a \cos(\delta) \dots (3)$$

$$E_f = V \cos(\delta) + I_{ad} X_{sd} \dots (4)$$

substitute from (3) into (2)

$$V \sin(\delta) = I_a X_{sq} \cos(\delta)$$

dividing both sides over cos(δ)

$$V \tan\delta = I_a X_{sq}$$

$$\tan\delta = \frac{I_a X_{sq}}{V}$$

V= 3300/√3 = 1905.256 V

$1.5\times10^6 = 3 \times 1905.256 \times I_a \times 1 \rightarrow I_a = 262.432$ A

As proved above

$$\tan\delta = \frac{I_a X_{sq}}{V} = \frac{262.432 \times 3}{1905.256} = 0.41322$$

δ = 22.45°

$I_{ad} = I_a \sin(\delta) = 262.432 \times \sin(22.45) = 100.223$ A

$$E_f = V \cos(\delta) + I_{ad} X_{sd} = 1905.256 \times \cos(22.45) + 100.223 \times 4 = 2161.75\, V$$

Since all losses are neglected, then the output mechanical power = input power

$$P_{out} = \frac{3\, V E_f}{X_{sd}} \sin(\delta) + \frac{3\, V^2 (X_{sd} - X_{sq})}{2\, X_{sd}\, X_{sq}}\, Sin(2\delta)$$

$$P_{out} = \frac{3 \times 1905.256 \times 2161.75}{4} \sin(\delta) + \frac{3 \times 1905.256^2 (4 - 3)}{2 \times 4 \times 3}\, Sin(2\delta)$$

$$P_{out} = 3089015.37 \sin(\delta) + 453750.1\, sin(2\delta)$$

at δ = 22.45 → $P_{out} = 1.5\times10^6$ as given rated power

To get the maximum power

$$\frac{dP_{out}}{d\delta} = 0$$

$$\frac{dP_{out}}{d\delta} = 3089015.37\cos(\delta) + \ 2 \times 453750.1\cos(2\delta) = 0$$

As we know

$\cos(2\delta) = 2\cos^2(\delta) - 1$

$$\frac{dP_{out}}{d\delta} = 3089015.37\cos(\delta) + \ 2 \times 453750.1 \times \{2\cos^2(\delta) - 1\} = 0$$

$$1815000.4\, cos^2(\delta) + 3089015.37\cos(\delta) - \ 907500.2 = 0$$

$$cos^2(\delta) + 1.7019365\cos(\delta) - \ 0.5 = 0$$

solving for $\cos(\delta) \rightarrow \cos(\delta) = 0.2554435$ (accepted) Or $\cos(\delta) = -1.95738$ (rejected)

$\delta = 75.2°$

to get the maximum power, substitute the value of δ obtained in power eqn.

$$P_{out} = 3089015.37\sin(75.2) + \ 453750.1\, sin(2 \times 75.2) = 3.211\, MW$$

<u>Example (5):</u>

A 3-phase, 400 V (line), Y-connected, salient-pole synchronous generator supplies a current with a phase angle of 30° lagging. If the load angle is 14°, calculate

a) the direct and quadrature components of armature current,
b) the no-load voltage and the percentage regulation.
c) The output power
d) If the load current is kept constant but the phase angle becomes 10° lagging, find the new values of the no-load voltage and the load angle.

Known that $X_{sd} = 10\ \Omega$ and $X_{sq} = 6.5\ \Omega$, armature resistance to be negligible.

$V_a = 400/\sqrt{3} = 230.9 \approx 231$ volt

Taking the voltage scale as each 20 V is represented by 1 cm

Therefore, V_a represented by the vector OA= 11.5 cm as shown in the phasor diagram,

The current vector OX1 is drawn at 30° lag the voltage vector OA

Draw the vector OQ at 14° to represent the quadrature axis and E_{f1} must lie on this line

Draw the vector AZ ⊥ OX1 and intersect the quadrature axis OQ at point B,

Vector AB = 3.9 cm = 78 volt that represent the voltage drop $I_a X_{sq}$

Then I_a = 78/6.5 = 12.0 A ------------------------- (1)

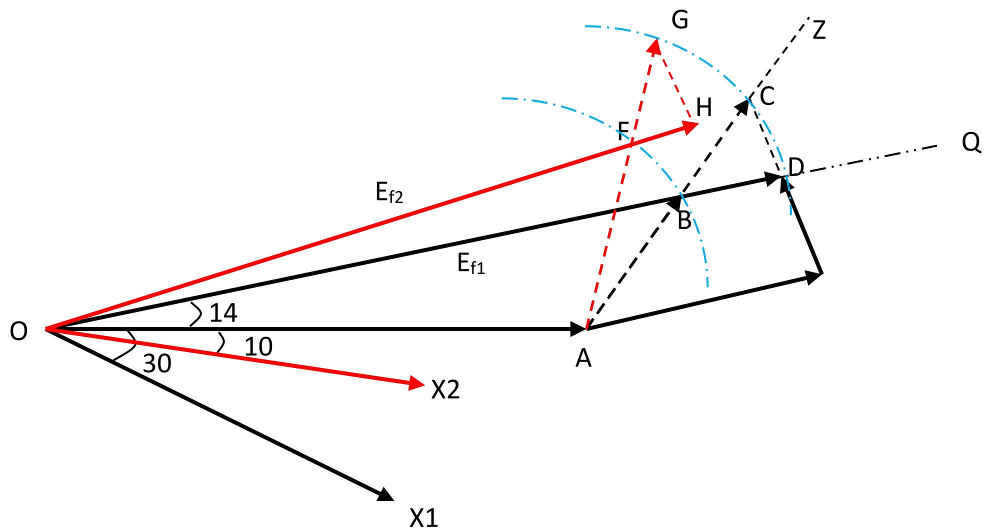
a) $I_{ad} = I_a \sin(14+30) = 8.34$ A ##

$I_{aq} = I_a \cos(14+30) = 8.63$ A ##

Draw the vector AC that represent $I_a X_{sd}$ = 12.0×10 =120 V = 6.0 cm

From point C draw CD ⊥ OQ, then the no-load voltage E_{f1} is represented by the vector OD = 15.3 cm = 306 V ##

Regulation = (306 – 231)/231 = 32.46% ##



The output power is given as

$$P_{out} = \frac{3\, V E_f}{X_{Sd}} \sin(\delta) + \frac{3\, V^2 (X_{Sd} - X_{Sq})}{2\, X_{Sd}\, X_{Sq}}\, Sin(2\delta)$$

$$P_{out} = \frac{3 \times \frac{400}{\sqrt{3}} \times 306}{10} \sin(14) + \frac{3 \times \frac{400}{\sqrt{3}}^2 (10 - 6.5)}{2 \times 65}\, Sin(28)$$

$$P_{out} = 5128.82 + 2022.34 = 7151.16\, W$$

Now if the angle between voltage and current is decreased to 10° lag and the armature current is kept constant at 12 A as obtained from eqn. (1)

Draw the vector AF = AB = 3.9 cm = 78 V and ⊥ the current vector OX2

Draw the vector AG = AC = 6 cm = 120 V and ⊥ the current vector OX2

Draw the vector OF that represent the new quadrature axis

The new load angle is FOA = 18° #

then the no-load voltage E_{f2} is represented by the vector OH = 13.8 cm = 276 V #

<u>Another solution (Analytical solution)</u>

From phasor diagram,

$I_{aq}X_{sq} = V_a \sin(\sigma)$

$$I_{aq} = \frac{\frac{400}{\sqrt{3}}\sin(14)}{6.5} = 8.6\ A$$

But $I_{aq} = I_a \cos(\phi+\sigma)$

$$I_a = \frac{8.6}{\cos(30+14)} = 11.96\ A$$

$I_{ad} = I_a \sin(\phi+\sigma) = 11.96 \cos(30+14) = 8.31$ A

$E_{f1} = V_a \cos(\sigma) + I_{ad}X_{sd} = (400/\sqrt{3}) \cos(14) + 8.31\times10 = 307.2$ V

3. Parallel Operation of Alternators

The operation of connecting an alternator in parallel with another alternator or with common bus-bars is known as ***synchronizing***. Generally, alternators are used in a power system where they are in parallel with many other alternators. It means that the alternator is connected to a live system of <u>constant voltage</u> and <u>constant frequency</u>. Often the electrical system, to which the alternator is connected, has already so many alternators and loads connected to it that no matter what power is delivered by the incoming alternator, the voltage and frequency of the system remain the same. In that case, the alternator is said to be connected to ***infinite*** bus-bars.

For proper synchronization of alternators, the following four conditions must be satisfied

1. The terminal voltage (effective) of the incoming alternator must be the same as bus-bar voltage. Using Voltmeters, the **field current** of the oncoming generator should be adjusted until its **terminal voltage is equal to the line voltage** of the running system.

2. The speed of the incoming machine must be such that its frequency (= *PN*/60) equals bus-bar frequency. Using a frequency meter, the prime mover speed is adjusted so that the frequency of the incoming machine is slightly higher than the system frequency.

3. The phase of the alternator voltage must be identical with the phase of the bus-bar voltage. This is done by either of the following two methods:

Alternately connect a small induction motor to the terminals of each of the 2 generators. If the motor rotates in the same direction each time, then the phase sequence is the same for both generators. If the motor rotates in opposite directions, then the phase sequences differ, and 2 of the phases of the incoming generator must be reversed.

Another way is using the 3 light bulb method, where the bulbs are stretched across the open terminals of the switch connecting the generator to the system (as shown in Fig. 6). As the phase changes between the 2 systems, the light bulbs first get bright (large phase difference) and then get dim (small phase difference). If all 3 bulbs get bright and dark together, then the systems have the same phase sequence. If the bulbs brighten in succession, then the systems have the opposite phase sequence, and one of the sequences of the incoming generator must be reversed.

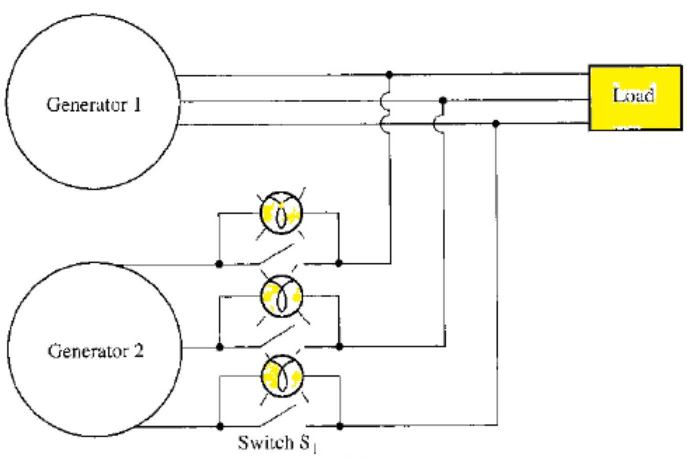


Fig. 6, Adjusting phase sequence using three lambs

4. The phase angle between identical phases must be zero.

Fine tuning the prime mover speed until the three lambs become dark for long time, It means that the switch must be closed and this is the proper time for synchronization.

To eliminate the element of personal judgment in routine operation of alternators, the machines are synchronized by a more accurate device called a synchronoscope as shown in Fig. 7. It consists of 3 stationary coils and a rotating iron vane which is attached to a pointer. Out of three coils, a pair is connected to one phase of the line and the other to the corresponding machine terminals, potential transformer being usually used. The pointer moves to one side or the other from its vertical position depending on whether the incoming machine is too fast or too slow. For correct speed, the pointer points vertically up.

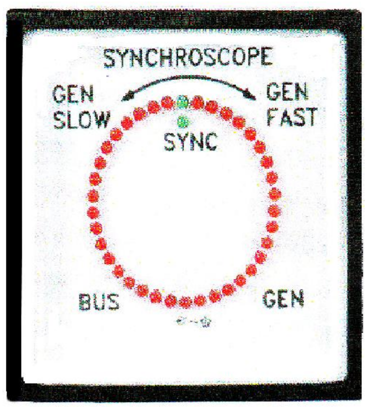


Fig. 7 Synchronoscope

Consider two alternators with identical speed/load characteristics connected in parallel as shown in Fig. 10. The common terminal voltage **V** is given by

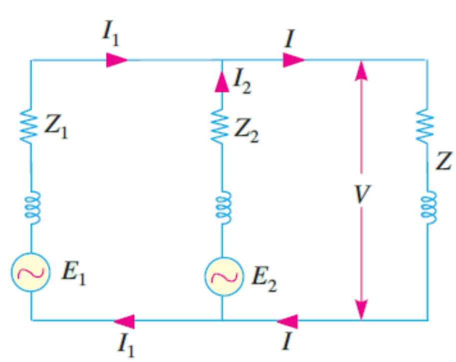


Fig. 10, Two synchronous generators operate in parallel

$$\mathbf{V} = \mathbf{E}_1 - \mathbf{I}_1\mathbf{Z}_1 = \mathbf{E}_2 - \mathbf{I}_2\mathbf{Z}_2$$

$$\therefore \quad \mathbf{E}_1 - \mathbf{E}_2 = \mathbf{I}_1\mathbf{Z}_1 - \mathbf{I}_2\mathbf{Z}_2$$

$$\text{Also} \quad \mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 \text{ and } \mathbf{V} = \mathbf{IZ}$$

$$\therefore \quad \mathbf{E}_1 = \mathbf{I}_1\mathbf{Z}_1 + \mathbf{IZ} = \mathbf{I}_1(\mathbf{Z}+\mathbf{Z}_1) + \mathbf{I}_2\mathbf{Z}$$

$$\mathbf{E}_2 = \mathbf{I}_2\mathbf{Z}_2 + \mathbf{IZ} = \mathbf{I}_2(\mathbf{Z}+\mathbf{Z}_2) + \mathbf{I}_1\mathbf{Z}$$

$$\therefore \quad \mathbf{I}_1 = \frac{(\mathbf{E}_1 - \mathbf{E}_2)\,\mathbf{Z} + \mathbf{E}_1\mathbf{Z}_2}{\mathbf{Z}\,(\mathbf{Z}_1 + \mathbf{Z}_2) + \mathbf{Z}_1\mathbf{Z}_2}$$

$$\mathbf{I}_2 = \frac{(\mathbf{E}_2 - \mathbf{E}_1)\,\mathbf{Z} + \mathbf{E}_2\mathbf{Z}_1}{\mathbf{Z}\,(\mathbf{Z}_1 + \mathbf{Z}_2) + \mathbf{Z}_1\mathbf{Z}_2};$$

$$\mathbf{I} = \frac{\mathbf{E}_1\mathbf{Z}_2 + \mathbf{E}_2\mathbf{Z}_1}{\mathbf{Z}\,(\mathbf{Z}_1 + \mathbf{Z}_2) + \mathbf{Z}_1\mathbf{Z}_2}$$

$$\mathbf{V} = \mathbf{IZ} = \frac{\mathbf{E}_1\,\mathbf{Z}_2 + \mathbf{E}_2\mathbf{Z}_1}{\mathbf{Z}_1 + \mathbf{Z}_2 + (\mathbf{Z}_1\mathbf{Z}_2/\mathbf{Z})};\ \mathbf{I}_1 = \frac{\mathbf{E}_1 - \mathbf{V}}{\mathbf{Z}_1};\ \mathbf{I}_2 = \frac{\mathbf{E}_2 - \mathbf{V}}{\mathbf{Z}_2}$$

The circulating current, I_c, under no-load condition is

$$\mathbf{I}_C = (\mathbf{E}_1 - \mathbf{E}_2)/(\mathbf{Z}_1 + \mathbf{Z}_2).$$

Example:

Two 3-phase alternator operating in parallel have induced e.m.fs on open circuit of 230 ∠ 0° and 230 ∠ 10° volts and respective reactances of j2 Ω and j3 Ω. Calculate ***(i)*** *terminal voltage* ***(ii)*** *currents and* ***(iii)*** *power delivered by each of the alternators to a load of impedance 6 Ω (resistive).*

$\mathbf{Z_1} = j\,2.0$, $\mathbf{Z_2} = j\,3.0$, $\mathbf{Z} = 6.0$; $\mathbf{E_1} = 230\angle 0°$ and $\mathbf{E2} = 230\angle 10° = 226.5 + j\,39.9$

$$(ii)\quad \mathbf{I}_1 = \frac{(E_1 - E_2)\,Z + E_1Z_2}{Z\,(Z_1 + Z_2) + Z_1Z_2} = \frac{[(230 + j0) - (226.5 + j39.9)] \times 6 + 230 \times j3}{6\,(j2 - j3) + j2 \times j3}$$

$$= 14.3 - j3.56 = 14.73\angle -14°$$

$$\mathbf{I}_2 = \frac{(E_2 - E_1)\,Z + E_2Z_1}{Z\,(Z_1 + Z_2) + Z_1Z_2} = \frac{(-3.5 + j39.9) + (222.5 + j39.9) \times j2}{6\,(j2 + j3) + j2 \times j3}$$

$$= 22.6 - j\,1.15 = 22.63\angle -3.4°$$

$$(i)\quad \mathbf{I} = \mathbf{I}_1 + \mathbf{I}_2 = 36.9 - j\,4.71 = 37.2\angle -7.3°$$

$$\mathbf{V} = \mathbf{IZ} = (36.9 - j4.71) \times 6 = 221.4 - j28.3 = 223.2\angle -7.3°$$

(iii) $P1 = V\,I_1\cos(\phi 1) = 223.2 \times 14.73 \times \cos(14-7.3) = 3265.283$ W/ph

$P2 = V\,I_2\cos(\phi 2) = 223.2 \times 22.63 \times \cos(7.3-3.4) = 5039.32$ W/ph

Example

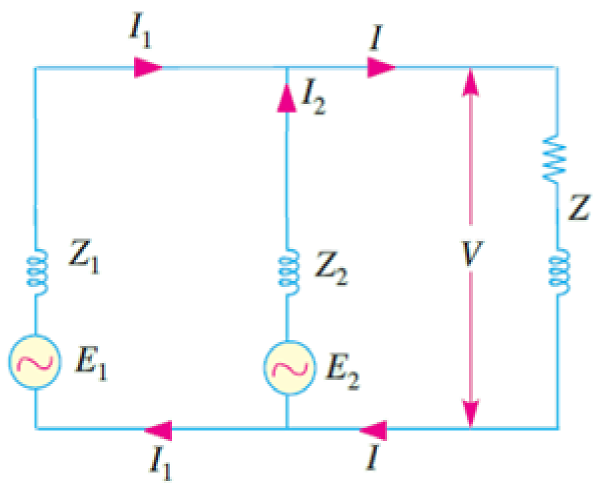
Two synchronous generators operate in parallel to deliver a total load of 36000 kW at 0.866 p.f. lag. Both machines are rated as follows:

Machine I: 30000 kVA, 13.2 kV, Y connected, Xs=0.775 pu

Machine II: 12250 kVA, 13.2 kV, Y connected, Xs= 1.0 pu

Determine for each machine:

the kW, the kVA, the kVAR, the current, the p.f, the power angle and emf.



the phase voltage = 13.2/√3 = 7.621 kV and is taken as a reference

For Load:

$$I = \frac{36000 \times 1000}{\sqrt{3} \times 13.2 \times 1000 \times 0.866} = 1818.2352\ A$$

$$I = 1818.2352\angle -30^{\circ}$$

For machine I:

$$I_{1(rated)} = \frac{30000 \times 1000}{\sqrt{3}\ 13.2 \times 1000} = 1312.16\ A$$

$$Z_{base1} = \frac{13.2 \times 1000/\sqrt{3}}{1312.16} = 5.808\ \Omega$$

$$X_{s1} = 0.775 \times 5.808 = 4.5012\ \Omega$$

For machine 2:

$$I_{2(rated)} = \frac{12250 \times 1000}{\sqrt{3}\, 13.2 \times 1000} = 535.799\, A$$

$$Z_{base1} = \frac{13.2 \times 1000/\sqrt{3}}{535.799} = 14.224\, \Omega$$

$$X_{s1} = 1.0 \times 14.224 = 14.224\, \Omega$$

Then we can calculate the actual current for the two generators as:

$$I_1 = I \times \frac{Z_2}{Z_1 + Z_2} = 1818.2352\angle -30 \frac{14.224\angle 90}{4.5012\angle 90 + 14.224\angle 90} = 1381.164\angle -30A$$

$$I_2 = I \times \frac{Z_1}{Z_1 + Z_2} = 1818.2352\angle -30 \times \frac{4.5012\angle 90}{4.5012\angle 90 + 14.224\angle 90} = 437.07\angle -30A$$

Since the synchronous impedance of both machines is pure reactance, the power factor of both machines is similar to that of the load. This means

$$p.f_1 = p.f_2 = p.f_{Load} = \cos(30) = 0.866\, lag$$

$$P_1 = \sqrt{3}\, 13.2 \times 1000 \times 1381.164 \times 0.866 = 27.245346\, MW$$

$$P_2 = \sqrt{3}\, 13.2 \times 1000 \times 437.07 \times 0.866 = 8.653732\, MW$$

As check if we add P_1 and P_2, we must obtain the load power

27.245346 + 8.653732 = 35.9 MW which is very near to 36 MW

$$\sin(\varphi 1) = \sin(\varphi 2) = \sin(30) = 0.5$$

$$Q_1 = \sqrt{3}\, 13.2 \times 1000 \times 1381.164 \times 0.5 = 15.788825\, MVAR$$

$$Q_2 = \sqrt{3}\, 13.2 \times 1000 \times 437.07 \times 0.5 = 4.996381\, MVAR$$

$$E_{f1} = I_1 Z_1 + V = 1381.164\angle -30 \times (4.5012\angle 90) + \frac{13.2 \times 10^3}{\sqrt{3}} = 12004.54\angle 26.65\, V$$

$$E_{f2} = I_2 Z_2 + V = 437.07\angle -30 \times (14.224\angle 90) + \frac{13.2 \times 10^3}{\sqrt{3}} = 12004.54\angle 26.65\, V$$

Example:

Two 3-phase, Y-connected alternators have the following ratings:

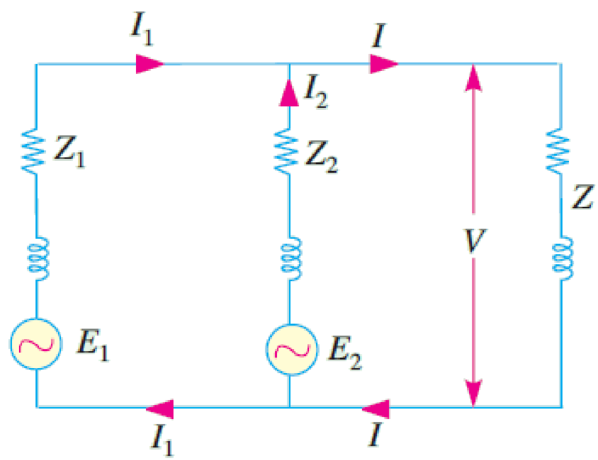
E_{f1} =1732 ∠*0*°V, E_{f2} = 1732 ∠*15*°V, R_{a1}=0.11Ω, R_{a2}=0.21Ω, X_{s1}=2.0Ω, X_{s2}=4.0Ω. The two machines supply a load of impedance 3+J2 Ω. Find

a) Terminal voltage

b) Load, machine #1 and machine #2 currents.

c) Power dissipated by the load.

d) Under no-load condition, find the circulating current.



Z1 = 0.11 + J 2 Ω (First Synchronous generator impedance)

Z2 = 0.21 + J 4 Ω (Second Synchronous generator impedance)

Z = 3 + J 2 Ω (Load impedance)

$$V = \frac{E_{f1}Z_2 + E_{f2}Z_1}{Z_1 + Z_2 + \frac{Z_1 Z_2}{Z}}$$

$$V = \frac{1732\,(0.21 + J4) + 1732\angle 15(0.11 + J2)}{0.11 + J2 + 0.21 + J4 + \frac{(0.11 + J2)(0.21 + J4)}{3 + J2}} = \frac{10329.1682\angle 91.9352}{7.55438\angle 100.5913}$$

$$= 1367.31\angle -8.66\ V$$

$$I = \frac{V}{Z} = \frac{1367.31\angle -8.66}{3 + J2} = 379.224\angle -42.35\ A$$

$$I_1 = \frac{E_{f1} - V}{Z_1}$$

$$I_1 = \frac{1732 - 1367.31\angle - 8.66}{(0.11 + J2)} = 215.8894\angle - 58.4215\ A$$

$$I_2 = \frac{E_{f2} - V}{Z_2}$$

$$I_1 = \frac{1732\angle 15 - 1367.31\angle - 8.66}{(0.21 + J4)} = 181.945\angle - 23.151\ A$$

The power dissipated by the load = 3 × I^2 × R_L = 3×$(379.224)^2$×3 =1.294297 MW

Under no-load condition, the circulating current is given as:

$$I_c = \frac{E_{f1} - E_{f2}}{Z_1 + Z_2}$$

$$I_c = \frac{1732 - 1732\angle 15}{0.11 + J2 + 0.21 + J4} = 75.25\angle - 169.447$$

Example

Two 3-ph, 6.6 kV, Y-connected, alternators supply a load of 3000 kW at 0.8 p.f. lagging. The synchronous impedance per phase of machine A is 0.5+J10 Ω and that of machine B is 0.4 +J12 Ω. The excitation of machine A is adjusted so that it delivers 150 A. The load is shared equally between the machines. Determine the current, p.f., induced e.m.f., and load angle of each machine.

Since the load is shared equally between the two alternators, then

the active power delivered P_A = P_B = 1500 kW

At load, the total current is

$$I = \frac{3000 \times 10^3}{\sqrt{3} \times 6.6 \times 10^3 \times 0.8} = 328.04\ A$$

The angle of total current is $\cos^{-1}$ 0.8 = 36.87° Lag

For Machine A, the current = 150 A, then

$$I_A = \frac{1500 \times 10^3}{\sqrt{3} \times 6.6 \times 10^3 \times \cos(\varphi_A)} = 150\ A$$

$$\cos(\varphi_A) = \frac{1500 \times 10^3}{\sqrt{3} \times 6.6 \times 10^3 \times 150} = 0.8748$$

$$\varphi_A = 28.982°$$

The p.f. of machine A is cos(28.982) = 0.875 lagging

As we know

$$I = I_A + I_B \rightarrow \; I_B = I - \; I_A$$

$$I_B = 328.04\angle - 36.87 - 150\angle - 28.982 = 180.64\angle - 43.414°$$

The p.f. of machine B is cos(43.414) = 0.726 lagging

$$E_{fA} = I_A Z_A + V = 150\angle - 28.982 \; \times (0.5 + \text{J}10) + \frac{6.6 \times 10^3}{\sqrt{3}} = 4776.46\angle 15.49 \, V$$

This means

E_{fA} = 4776.46 V and the load angle σ_A = 15.49°

By the same way for machine B

$$E_{fB} = I_B Z_B + V = 180.64\angle - 43.414 \; \times (0.4 \; + \text{J}12) + \frac{6.6 \times 10^3}{\sqrt{3}} = 5565.76\angle 15.9 \, V$$

This means

E_{fB} = 5565.76 V and the load angle σ_B = 15.9°

Sheet 3 (Salient-pole Synchronous Machines)

1- *On a certain salient-pole synchronous generator, X_{sd} = 0.9 pu and X_{sq}=0.6 pu. The machine is operating at full load, 0.8 p.f. lag. Calculate the value of excitation voltage in terms of the terminal voltage. Calculate also the load angle and the direct and quadrature component of current. Neglect R_a.*

[16.9 pu, 19.4°, 0.833 pu and 0.555 pu]

2- *A 20 MVA, 3-phase, star-connected, 50-Hz, salient-pole has Xd = 1 p.u.; Xq = 0.65 p.u. and Ra = 0.01 p.u. The generator delivers 15 MW at 0.8 p.f. lagging to an 11-kV, 50-Hz system. What is the load angle and excitation e.m.f. under these conditions?* *[18°; 1.73 p.u]*

3- *A salient-pole synchronous generator delivers rated kVA at 0.8 p.f. lagging at rated terminal voltage. It has Xd = 1.0 p.u. and Xq = 0.6 p.u. If its armature resistance is negligible, compute the excitation voltage under these conditions.*

[1.77 p.u]

4- *A 20-kVA, 220-V, 50-Hz, star-connected, 3-phase salient-pole synchronous generator supplies load at a lagging power factor angle of 45°. The phase constants of the generator are Xd = 4.0 Ω ; Xq = 2 Ω and Ra = 0.5 Ω. Calculate (i) power angle and (ii) voltage regulation under the given load conditions.*

[(i) 20.6° (ii) 142%]

5- *A 3-phase salient-pole synchronous generator has Xd = 0.8 p.u.; Xq = 0.5 p.u. and Ra = 0. Generator supplies full-load at 0.8 p.f. lagging at rated terminal voltage. Compute (i) power angle and (ii) no-load voltage if excitation remains constant.*

[(i) 17.1° (ii) 1.6 p.u]

6- *A 3-phase alternator has a direct-axis synchronous reactance of 0.7 p.u. and a quadrature axis synchronous reactance of 0.4 p.u. Draw the vector diagram for full-load 0.8 p.f. lagging and obtain **(i)** the load angle and **(ii)** the no-load voltage.*

[(i) 16.5° (ii) 1.553 p.u]

7- *A 3-phase, star-connected, 50-Hz synchronous generator has direct-axis synchronous reactance of 0.6 p.u. and quadrature-axis synchronous reactance of 0.45 p.u. The generator delivers rated kVA at rated voltage. Draw the phasor diagram at full-load 0.8 p.f. lagging and hence calculate the open-circuit voltage and voltage regulation. Resistive drop at full-load is 0.015 p.u.* [15.3° & 1.448 p.u & 44.8%]

8- *A 3-phase, Y-connected syn. generator supplies current of 10 A having phase angle of 20° lagging at 400 V. Find the load angle and the components of armature current Id and Iq if Xd = 10 ohm and Xq = 6.5 ohm. Assume armature resistance to be negligible. Then calculate the no-load voltage and the percentage regulation.*

[8.23° & 4.73A, 8.81A & 443V, 10.75%]

9- *Two three-phase, 6.6 kV, Y-connected, alternators supply a load of 3000 kW at 0.8 p.f. lagging. The synchronous impedance per phase of machine A is 0.5+j10 Ω and of machine B is 0.4+j12 Ω. The excitation of machine A is adjusted so that it delivers 150 A at a lagging p.f. The load is shared equally between the two machines. Determine the current, p.f., induced emf and load angle of each machine.*

10- *Two synchronous generators operate in parallel to deliver a total load of 36000 kW at 0.866 p.f. lag. Both machines are rated as follows:*

Machine I: 30000 kVA, 13.2 kV, Y connected, Xs=0.775 pu

Machine II: 12250 kVA, 13.2 kV, Y connected, Xs= 1.0 pu

Determine for each machine:

the kW, the kVA, the kVAR, the current, the p.f, the power angle and emf.